- **Heaviside Step Function**

  - $H(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & x = 0 \\ 1 & x > 0 \end{cases}$ Shifted: $H(x-a) = \begin{cases} 0 & x < a \\ \frac{1}{2} & x = a \\ 1 & x > a \end{cases}$

  - Physical meaning: jumps from 0 to 1 at some instant time.
  - Applications: turning a switch or signal from off to on.
    - $1 - H(x)$ represents turning a switch from on to off.

- **Sign Function**

  - $\text{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$ Shifted: $\text{sgn}(x-a) = \begin{cases} -1 & x < a \\ 0 & x = a \\ 1 & x > a \end{cases}$

  - $\text{sgn}(x) = \dfrac{|x|}{x}$ $\text{sgn}(x) = 2H(x) - 1$

  - Returns the sign of a real number.
  - Physical meaning: represents the reversal of something

- **Dirac Delta Function**

  - $\delta(x) = \begin{cases} 0 & x < 0 \\ \infty & x = 0 \\ 0 & x > 0 \end{cases}$ Shifted: $\delta(x-a) = \begin{cases} 0 & x < a \\ \infty & x = a \\ 0 & x > a \end{cases}$

  - $\delta(x) = \dfrac{d}{dx} H(x)$ $\delta(x) = \dfrac{1}{2}\dfrac{d}{dx} \text{sgn}(x)$

  - Properties: $\displaystyle\int_{-\infty}^{\infty} \delta(x) = 1$ ($\delta(x)$ is a unit impulse) $\displaystyle\int_{-\infty}^{\infty} f(x)\delta(x-a) = f(a)$

  - Physical meaning: any instantaneous unit impulse – striking of an object with a hammer, the jump in the current when a switch is flipped on.

- **Boxcar Function**

  - $\Pi_{a,b}(x) = H(x-a) - H(x-b)$

  - Physical meaning: represents something being on from time a to b.

  - **Rectangular Function**: $\Pi(x) = \begin{cases} 0 & |x| > \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \\ 1 & |x| < \frac{1}{2} \end{cases}$

- **Ramp Function**

  - $R(x) = \begin{cases} 0 & x < 0 \\ x & x \geq 0 \end{cases}$ Shifted: $R(x-a) = \begin{cases} 0 & x < a \\ x-a & x \geq a \end{cases}$

  - $R(x) = xH(x)$ $H(x) = \dfrac{d}{dx} R(x)$ $R(x) = H(x) * H(x)$ (convolution)